

For 150 years the 40 metre high Nelson Pillar stood in the middle of Dublin's O'Connell Street until it was demolished in 1966 after being damaged by an explosion. In 1998 Dublin City Council launched an international architectural competition for a replacement monument. There were 205 entries and the winning design was an elegant spire by Ian Ritchie Architects. It is a 120 m high cone that is three metres wide at the base and tapers to 150 mm at the top. Although it is essentially art, architecture and sculpture, it is also, as we will look at in this lesson, engineering, science and technology.

Fig.1 The Spire



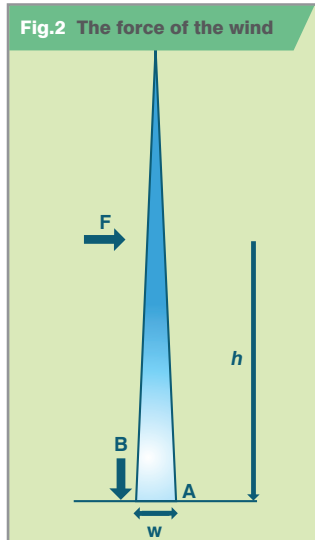
How was the spire designed?

The spire was manufactured in eight sections of up to 20 m in length each of which is a *frustum* – a truncated cone. The sections were made from rolled sheets of *stainless steel*. A thick *flange* with holes in it was *welded* at the ends of adjoining sections to enable them to be *bolted* together. The top two frusta are joined by a threaded connector.

What effect has wind?

The force of the wind must be taken into account in the design of tall structures. The *moment of a force*, i.e. its turning effect, is the magnitude of the force multiplied by the perpendicular distance from its line of action to the point about which it can turn (the *fulcrum*). The force is a function of the wind speed and the surface area on which it acts.

For example, if a force F due to wind acts at a height h then the moment of the force around the point A is $F \times h$. The structure is held by bolts at the base. The moment, around A, of the downward force B supplied by the bolts at the base is equal to $B \times w$. This must be sufficient to balance $F \times h$ if the structure is to remain standing. Because the diameter of the base is just 3 m and the height is so great, very large forces are required to withstand the buffeting of the wind. The bolts attaching the spire to the ground and those joining the sections are tightened to such a degree that the force they supply is always greater than the bending force caused by the wind.

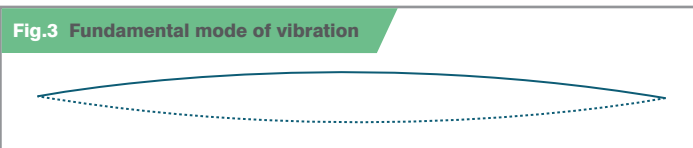


The force of the wind on a structure depends on a number of factors: the shape of the structure, the height above the ground, nearby structures and the wind speed. The force (given in *newtons*) on each square metre on a vertical surface varies from about 0.4 to 0.9 times the square of the wind speed (given in m/s) approximately – where the higher value applies to height above about 40 m. For example in a 30 m/s wind the horizontal force on the Spire is of the order of 80,000 N or about 8000 kg weight.

Resonance

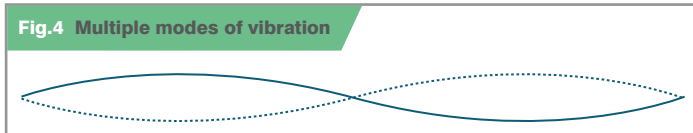
A less obvious but more significant effect is that of *resonant vibration*. Almost all structures can vibrate and if they vibrate uncontrollably they can collapse. When an *open guitar string* is strummed it usually vibrates at its lowest frequency and produces its lowest note. In this *fundamental mode* of vibration the length of the string is equal to one half-wave and there is a fixed point or *node* at each end.

Fig.3 Fundamental mode of vibration



It can however be forced to vibrate in other modes with frequencies that are *integral multiples* of its fundamental frequency. If the exact centre of the string is lightly touched while the string is plucked it can easily be set vibrating at twice the fundamental frequency; in this instance two half waves can be observed by looking along the string. Looking along the vibrating string a still point or node will be visible at the centre as well as at each end. It is also possible to set a guitar string vibrating at three times the fundamental frequency by touching it lightly at a point that is a third of its length from one end while it is being plucked. In a similar way other *harmonics* can be formed.

Fig.4 Multiple modes of vibration

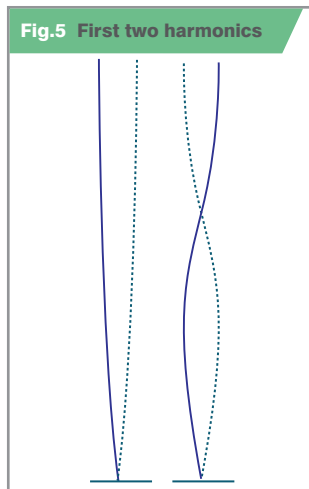


Resonance occurs when an external force is applied at a frequency that matches a natural frequency of vibration of a structure; at the resonant frequency the *amplitude* of the vibrations can become very large and destructive. In the case of metal structures, prolonged and large alternating stresses can cause *metal fatigue*.

How can a steady wind induce vibrations?

The answer to this question is too complex to analyse here but some common instances of this phenomenon may help. Blowing into a whistle or across the mouth of a bottle can produce notes of definite frequencies. As a steady flow of air passes an obstacle little swirls or *eddies* are usually produced; these have a *rotational frequency* of their own and can induce vibration in the obstacle that caused them to form. Similar eddy currents are commonly seen when water flows around stones and rocks in a river.

Fig.5 First two harmonics



Tuned Mass Damping

The Spire is fixed at the bottom and free at the top so the possible modes of vibration would be expected to have a node at the bottom and an *anti-node* (site of maximum movement) at the free end. If it were a uniform structure then the first two modes of vibration would be expected to be as shown. In the second of these the wavelength is a third of the wavelength of the fundamental and so its frequency is three times the fundamental. In the case of the Spire, the calculated fundamental frequency of vibration is 0.274 Hz. The second, third and fourth modes of vibration have theoretical frequencies of 0.839 Hz, 1.730 Hz and 2.820 Hz respectively.

Resonance may be desirable in musical instruments but is generally undesirable in large structures. In the case of the Spire two large masses are suspended on cables inside of the 5th section in order to suppress resonant vibration. By varying the length of these cables the resonant frequency of the masses is adjusted or *tuned*. Their frequencies are set to correspond to the first two modes of vibration of the Spire. If the Spire moves in one direction the masses move the opposite direction. '*Viscous dampers*' (not unlike car shock absorbers) connect the masses to the inside surface of the Spire. The vibrational energy is dissipated as heat in the viscous dampers. The upper mass is 1250 kg and the lower one is 800 kg.

Resonance at a few higher frequencies is also significant but the thousands of holes in the top section dampen these and also allow the lights to be seen.

Dublin City Council

Dublin City Council is the democratic body governing Dublin City. It is the largest Local Authority in the country with a staff of 6,200 approx and it provides a wide range of services for its citizens and businesses.

The key executive in the council is the City Manager who has overall responsibility for all functions of the council. The Council also has an elected assembly of 52 members, elected every five years. The Lord Mayor is elected annually by the Assembly to act as its chairman and as the symbolic head of the city.

The Council has a number of committees focusing on the development of policy in six major areas of Council activity. One third of the members are from outside the Council and represent, for example, social partner, business or community interests. There are also five committees focusing on the needs of specific areas.

The principal Council services (by expenditure) are environmental protection, housing and building, water supply and sewerage and road transportation and safety.

Dublin City covers an area of 11,500 hectares and has a population of 500,000. The total annual Council expenditure is nearly €800m.

A key element of the Development Plan for Dublin City is the implementation of Integrated Area Plans (IAP's). The O'Connell Street Integrated Area Plan was the first of the IAP's to be developed in 1998 when it was recognised that the main street of the capital was in decline.

The plan set out a number of objectives for improving the street. The first project was a replacement for Nelson's Pillar, the Spire. Although it was a controversial project, it is now recognised as a very important symbol of Dublin city and its completion marked a turning point in the rejuvenation of O'Connell Street.

The Spire could not have been constructed without the design of Ian Ritchie Architects and Ove Arup Engineering Consultants supported by Davis Langdon Everett as Quantity Surveyors.

Syllabus Reference

Leaving Certificate Physics

Waves - Properties of waves; Wave phenomena

Vibrations and Sound – Resonance; Vibrations in strings and pipes

Junior Certificate Science

Section 3B5 – Sound

Learning Objectives

On completion of this lesson students should:

- Understand that structures can vibrate
- Understand what is meant by resonance
- Appreciate that resonance is the basis of music instruments
- Understand that harmonics are integral multiples of a fundamental frequency
- Have some understanding of how a steady air flow can cause resonant vibrations to build up
- Appreciate how vibrations may be suppressed by using tuned mass dampers
- Appreciate the physical and technical challenges that had to be met in erecting the Spire.

General Learning Points

- The Spire of Dublin was designed in 1998 to replace the Nelson Pillar which was demolished in 1966, 150 years after it was erected.
- Most structures have a natural frequency of vibration. If they are subjected to recurring forces at that particular frequency large vibration may be induced.
- Resonant vibrations of strings, columns of air, blocks, reeds etc. are the source of sound in most musical instruments.
- Masses whose resonant frequency is tuned to match that of a vibrating structure can be used to suppress or dampen vibrations in the structure.
- The moment of a force, also known as the turning effect of a force, is the force applied multiplied by the perpendicular distance from the fulcrum.
- The force of the wind on a structure depends on the shape of the structure, its height above the ground, the location of nearby structures and the speed of the wind.

Activities

Mandatory Practical Activities

Investigation of the variation of fundamental frequency of a stretched string with length.

Investigation of the variation of fundamental frequency of a stretched string with tension.

True or False

Indicate whether the following are true (T) or false (F) by drawing a circle around T or F.

- | | | |
|---|---|---|
| (a) A frustum is the fixed point on which a lever can rotate | T | F |
| (b) The moment of a force is also the turning effect of the force | T | F |
| (c) The force of wind on a structure depends solely on wind speed | T | F |
| (d) The suppression of resonant vibrations is called amplification | T | F |
| (e) Resonant vibrations can be induced in metal rods and strings but not in columns of air. | T | F |
| (f) Harmonics are integral multiples of a fundamental frequency. | T | F |
| (g) Nodes are points at which the amplitude of vibration is a maximum. | T | F |
| (h) The total mass of the two tuned mass dampers in the Spire is about two tonnes. | T | F |
| (i) For a column that is fixed at one end the first two harmonics are in the ratio of 1:3 | T | F |
| (j) A steady wind can induce resonance. | T | F |

Check your answers to these questions on www.sta.ie

Examination Questions

2006 Leaving Certificate Physics Higher Level

How does resonance occur in an acoustic guitar?

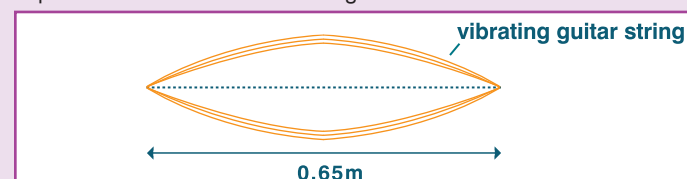
What is the relationship between frequency and tension for a stretched string?

A stretched string of length 80 cm has a fundamental frequency of vibration of 400 Hz. What is the speed of the sound wave in the stretched string?

2005 Leaving Certificate Physics Higher Level

Describe an experiment to demonstrate that sound is a wave motion.

Sound travels as longitudinal waves while light travels as transverse waves. Explain the difference between longitudinal and transverse waves.



The diagram shows a guitar string stretched between supports 0.65 m apart. The string is vibrating at its first harmonic. The speed of sound in the string is 500 m s⁻¹. What is the frequency of vibration of the string?

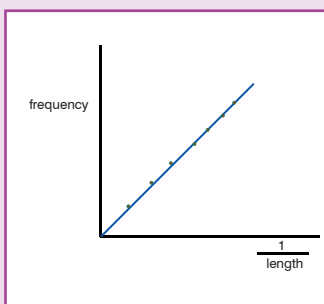
Draw a diagram of the string when it vibrates at its second harmonic.

What is the frequency of the second harmonic?

2004 Leaving Certificate Physics Ordinary Level

A student investigated the variation of the fundamental frequency f of a stretched string with its length l .

Draw a labelled diagram of the apparatus used in this experiment. Indicate on the diagram the points between which the length of the wire was measured.



The student drew a graph, as shown, using the data recorded in the experiment, to illustrate the relationship between the fundamental frequency of the string and its length.

State this relationship and explain how the graph verifies it.

The student then investigated the variation of the fundamental frequency f of the stretched string with its tension T . The length was kept constant throughout this investigation.

How was the tension measured?

What relationship did the student discover?

Why was it necessary to keep the length constant?

How did the student know that the string was vibrating at its fundamental frequency?

2006 Junior Certificate Higher Level

The Millennium Spire, in Dublin, is made from steel.

Iron and steel show visible signs of corrosion.

Give one visible sign of corrosion.

2004 Junior Certificate Higher Level

Question 1(g)

The diagram shows two waves travelling with the same velocity.

Which wave has the higher frequency?

Wave _____

Give a reason for your answer.

Reason _____



For further examples of past paper questions check www.sta.ie

Did You Know?

- The Dublin Spire is 120 metres high, has a mass of 126 tonnes and has a surface area of 600 m².
- The length of the internal weld is 3 kilometres, and contains 204 structural bolts.
- The skin is made up of stainless steel plate ranging in thickness from 35 mm at the base, 20 mm in the centre and tapers to 10 mm at the top of the structure.
- The sway at the top of the Spire can be up to 1.5 m in any direction.
- If the rim of a wine glass is gently stroked the resonant vibrations are set up and a characteristic note is heard.
- Nelson's Pillar, which was 41 m high, was erected in 1808 and remained the most distinctive landmark in Dublin until it was demolished in 1966. It was funded by public subscription and cost £6,856.
- The original Tacoma Narrows Bridge, New Jersey USA, was opened on 1st July 1940. It was known to oscillate strongly even in moderate winds. It disintegrated on the 7th November the same year due to extreme resonance.

Biographical Notes

Ernst Chladni (1756-1827)

Chladni investigated resonant vibrations in metal plates. A stem was added to the centre of a plate and it was then fixed horizontally in a strong stand. The plate was sprinkled with fine powder and then bowed using a violin bow. Depending on the shape of the plate and the point at which it was bowed different patterns were formed in the powder; it moved from the points where the amplitude of the vibration was greatest (antinodes) to the points of minimum amplitude (nodes).



Read more about these and other famous scientists at www.sta.ie

Revise the Terms

Can you recall the meaning of these terms? Reviewing the terminology is a powerful aid for recall and retention.

Frustum; stainless steel; flange; welded; bolted; moment of a force; fulcrum; newtons; resonant vibration; open guitar string; fundamental mode; node; integral multiples; harmonics; amplitude; metal fatigue; eddies; rotational frequency; anti-node; tuned; Viscous dampers.

Check the glossary of terms for this lesson at www.sta.ie